KINETIC MODEL OF GAS FLOW IN A POROUS BODY

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UDC 533.7:536.423

Mass transport in the process of gas evaporation and filtration through a porous body is studied using methods of the kinetic theory of gases. Theoretical results are compared with experiment.

The flow of gases through highly porous media is often described by the "dust gas" model [1-3], in which the real porous body is replaced by a homogeneous system of immobile spherical particles arrayed in unordered fashion. Gas filtration through such a medium is considered as mutual diffusion of the two gases, with the molecules of the one component, the body skeleton, having a size and mass significantly greater than the corresponding values for the real gas molecules. As a rule, published studies using this theory have considered the kinetic theory of gas flow in an infinite porous body, i.e., without consideration of the effects of coupling the porous body to an external medium. An expression for gas flow applicable over the entire range of Knudsen numbers can then be written in the form of a superposition of Knudsen and viscous flows. We will note that is then necessary to introduce empirical coefficients.

Kinetic theory methods have also been used to describe transport phenomena in dispersed media [4-9]. Using the method proposed in [10], [9] obtained a system of aerodynamic equations for dispersed media. In [6] the dynamics of a gas suspension were studied for the case of a free molecular flow over particles, while [7] found a closed system of kinetic equations for a fluidized bed. We will note that within the framework of the theory developed in [7], one cannot directly transform to the case of flow in a porous medium, since the kinetic equation for the particles then decays, so that it is necessary to introduce additional expressions to describe the gas-particle interaction.

The present study will use the dust gas model and the kinetic equation to formulate and solve the boundary problem of the kinetic theory of gases for mass transport in a porous body of finite thickness with consideration of particle resistance, i.e., the convective flow component. We will consider slow isothermal flow of a vapor through a porous layer of thickness L, bounded on the one side by the surface of the evaporating liquid, and on the other by liquid vapor, at a pressure p_L, such that $X = L$ ($p_L < p_0$) is known (Fig. 1). In this problem there are three characteristic dimensions: the porous body thickness L, the particle radius R, and the mean pore clearance λ_d [11], which is the mean molecular free path length with respect to the immobile spheres $\left(\lambda_d = \frac{4}{3}\frac{\varepsilon}{1-\varepsilon}R\right)$. Thus, the flow is characterized by two Knudsen numbers: $Kn = \lambda/R$ and $Kn_d = \lambda_d/R$, which may be of differing orders of magnitude.

For the gas molecule distribution function f within the porous body we write an equation with consideration of the "external" force F, which is the resultant of the collective interaction of gas molecules with particles:

$$\xi_x \frac{\partial f}{\partial X} + \frac{F}{m}\frac{\partial f}{\partial \xi_x} = 0. \qquad (1)$$

As a result, the kinetic problem of gas flow in a porous medium with complex gas–solid boundary is replaced by a one-dimensional problem, so that within the kinetic equation in place of the collision integral we introduce a term with the "external" force F, defined in the following manner. From the solution of the kinetic theory problem of gas flow over an individual spherical particle we calculate the force F_d, which acts on this particle in the case of diffuse reflection of the gas molecule from its surface [12]:

A. V. Lykov Heat and Mass Transfer Institute, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 46, No. 6, pp. 905-913, June, 1984. Original article submitted March 28, 1983.

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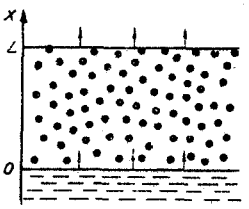

Fig. 1. Problem geometry.

$$F_d = F_S A_1(\mathrm{Kn}) = 3\pi\lambda\rho R\bar{\xi}vA_1(\mathrm{Kn}), \tag{2}$$

where

$$A_1(\mathrm{Kn}) = \frac{15 - 3\mathrm{Kn} + 3(8+\pi)\,\mathrm{Kn}^2}{15 + 12\mathrm{Kn} + 18\mathrm{Kn}^2 + 54\mathrm{Kn}^3}.$$

Then the value of F can be obtained from the expression

$$\varepsilon nF = -n_d F_d, \tag{3}$$

where

$$n_d = (1-\varepsilon)/\frac{4}{3}\pi R^3. \tag{4}$$

We then approximate the desired distribution f in the boundary problem in question in a two-sided Maxwell distribution

$$f = \begin{cases} f_1 = n_1\left(\dfrac{m}{2\pi kT}\right)^{3/2}\exp\left\{-\dfrac{m}{2kT}\left[(\xi_x - v_1)^2 + \xi_y^2 + \xi_z^2\right]\right\}, & \xi_x > 0, \\ f_2 = n_2\left(\dfrac{m}{2\pi kT}\right)^{3/2}\exp\left\{-\dfrac{m}{2kT}\left[(\xi_x - v_2)^2 + \xi_y^2 + \xi_z^2\right]\right\}, & \xi_x < 0 \end{cases} \tag{5}$$

and define n_1, n_2, v_1, v_2 using four moment equations of the form [13]

$$\frac{d}{dX}\int\xi_x\varphi f\,d\xi - \frac{F}{m}\int f\frac{\partial\varphi}{\partial\xi_x}\,d\xi = 0. \tag{6}$$

For φ we choose 1, ξ_x, ξ^2, ξ_x^3. It follows, in particular, from Eq. (6) that $\varphi = 1$ $\int\xi_x f\,d\xi = \mathrm{const}$, i.e., the gas velocity in the porous body is constant in the linear approximation and equal to

$$v = \frac{1}{n_0}\int\xi_x f\,d\xi = C'. \tag{7}$$

We now transform to the dimensionless quantities $x = X/L$ and ν_i, u_i $(i = 1, 2)$, c_j $(j = x, y, z)$, where $\nu_i = (n_i - n_0)/n_0$, $u_i = (m/2kT)^{1/2}v_i$, $c_j = (m/2kT)^{1/2}\xi_j$. Using the φ values presented above, in the linear approximation we obtain from Eqs. (5), (6)

$$\begin{gathered} \nu_1 + \pi^{1/2}u_1 - \nu_2 + \pi^{1/2}u_2 = C, \\ \frac{d}{dx}\left(\nu_1 + \frac{4}{\pi^{1/2}}u_1 + \nu_2 - \frac{4}{\pi^{1/2}}u_2\right) + \frac{AC}{\varepsilon} = 0, \\ \nu_1 + \frac{5\pi^{1/2}}{4}u_1 - \nu_2 + \frac{5\pi^{1/2}}{4}u_2 = D, \\ \frac{d}{dx}\left(\nu_1 + \frac{16}{3\pi^{1/2}}u_1 + \nu_2 - \frac{16}{3\pi^{1/2}}u_2\right) + \frac{AC}{\varepsilon} = 0, \end{gathered} \tag{8}$$

where

$$C = C'\left(\frac{2\pi m}{kT}\right)^{1/2}. \tag{9}$$

We find A from the expression $F = -\dfrac{kT}{2L\varepsilon}AC$, which follows from Eqs. (2)-(4), (7):

$$A = \frac{9(1-\varepsilon)}{\pi}\frac{L}{R}\mathrm{Kn}A_1(\mathrm{Kn}). \tag{10}$$

Solving system (8), we find

$$\nu_1 = -\frac{AC}{2\varepsilon}x + \frac{5}{2}C - 2D + \frac{E}{2}, \quad \nu_2 = -\frac{AC}{2\varepsilon}x - \frac{5}{2}C + 2D + \frac{E}{2},$$
$$u_1 = \frac{2}{\pi^{1/2}}(D-C) + \frac{B}{2}, \quad u_2 = \frac{2}{\pi^{1/2}}(D-C) - \frac{B}{2}. \tag{11}$$

The constants B, C, D, E are defined from the boundary conditions at x = 0 and x = 1, which in turn are obtained by calculating certain moments of the one-sided distribution functions on the boundary. These functions can be written in the following manner:
at x = 0

$$f_1 = n_0[\varepsilon + (1-\varepsilon)(1+\nu_{r1})]\left(\frac{m}{2\pi kT}\right)^{3/2}\exp\{-c^2\}, \quad c_x > 0, \tag{12}$$

at x = 1

$$f_2 = \varepsilon f_L + (1-\varepsilon)n_0(1+\nu_{r2})\left(\frac{m}{2\pi kT}\right)^{3/2}\exp\{-c^2\}, \quad c_x < 0, \tag{13}$$

where the quantities ν_{ri} are defined from the condition of impermeability of the solid particles:

$$\nu_{r1} = \nu_2 - \pi^{1/2}u_2, \tag{14}$$
$$\nu_{r2} = \nu_1 + \pi^{1/2}u_1, \tag{15}$$

and f_L is a Maxwellian function with parameters n_L, u_L (where u_L is the unknown dimensionless gas velocity above the porous body).

It follows from Eq. (12) with consideration of Eqs. (5), (14), that at x = 0

$$\nu_1 = (1-\varepsilon)(\nu_2 - \pi^{1/2}u_2), \quad u_1 = 0. \tag{16}$$

We find yet two more boundary conditions by calculating the moments of Eq. (13) at x = 1 relative to c_x and $c_x c^2$ and using Eq. (15):

$$(1-\varepsilon)(\nu_1 + \pi^{1/2}u_1) - (\nu_2 - \pi^{1/2}u_2) + \varepsilon(\nu_L - \pi^{1/2}u_L) = 0,$$
$$(1-\varepsilon)(\nu_1 + \pi^{1/2}u_1) - \left(\nu_2 - \frac{5\pi^{1/2}}{4}u_2\right) + \varepsilon\left(\nu_L - \frac{5\pi^{1/2}}{4}u_L\right) = 0, \tag{17}$$

where $\nu_L = (n_L - n_0)/n_0$.

Substituting Eq. (11) in Eqs. (16), (17), we obtain a system of four linear algebraic equations for determination of B, C, D, E, solution of which allows us to find ν_i, u_i from Eq. (11). We thus determine the gas velocity in the porous body from Eqs. (7), (9), and its density:

$$u = \frac{C}{2\pi^{1/2}}, \quad n = n_0(1+\nu) = \int f d\xi = n_0\left(1 + \frac{\nu_1}{2} + \frac{u_1}{\pi^{1/2}} + \frac{\nu_2}{2} - \frac{u_2}{\pi^{1/2}}\right). \tag{18}$$

Thus for the final solution of the conjugate kinetic problem for a specified n_L it is necessary to determine the velocity u_L. To do this we use the law of conservation of gas mass upon transition through the boundary x = 1, i.e., the equality of gas flows in the porous body and the external medium: $\frac{n_L}{n_0}u_L = \varepsilon\frac{n}{n_0}u$. The presence of the ε in this expression is explained by the fact that when the kinetic equation is used the macroscopic gas parameters are defined per unit volume of the entire porous body, and the portion of the volume occupied by the particles is considered only in Eq. (3) for the force F. Therefore, in calculating the real gas flow through a unit surface within the porous body it is necessary to multiply n/n_0 by ε, i.e., in the linear approximation

$$u_L = \varepsilon u = \frac{\varepsilon C}{2\pi^{1/2}}. \tag{19}$$

We note that in the present simplified formulation the Knudsen layer on the boundary x = 1 with the external medium is not considered. It may be considered by using the conservation laws in the gaseous phase. The solution of such a problem, which is obtained by numerical methods, will not be presented here.

TABLE 1. Quantities u_L in $[n(1) - n_L]/(n_0 - n_L)$ vs Kn for Evaporation from Porous Body with $\varepsilon = 0.9$, $L/R = 100$; $(n_L - n_0)/n_0 = -0.1$.

Kn	∞	500	100	50	30	20	10
$u_L \cdot 10^2$	0,239	0,239	0,240	0,241	0,242	0,244	0,248
$\frac{n(1)-n_L}{n_0-n_L}$	0,0221	0,0221	0,0222	0,0223	0,0224	0,0225	0,0230
Kn	5	1	0,5	0,1	0,01	0,001	0,0001
$u_L \cdot 10^2$	0,256	0,314	0,412	1,139	2,732	3,221	3,281
$\frac{n(1)-n_L}{n_0-n_L}$	0,0238	0,0292	0,0377	0,105	0,252	0,298	0,303

The presence of a nonequilibrium layer at the boundary x = 1 on the porous body side leads, in particular, to a discontinuity in gas density $n(1) - n_L$. On the basis of Eqs. (18), (11), (19) we write an equation for $[\nu(1) - \nu_L]/\nu_L = [n(1) - n_L]/(n_L - n_0)$:

$$\frac{\nu(1)-\nu_L}{\nu_L} = -1 + \frac{\pi^{1/2} u_L}{\varepsilon \nu_L}\left[1 + \frac{\varepsilon^2}{2}\left(1 - \frac{2}{\pi}\right) - \frac{2+A}{\varepsilon}\right]. \tag{20}$$

This density (pressure) change is an analog of the pressure loss on exit from a porous body in [14]. In contrast to the changes in macroscopic quantities in the kinetic theory of gases, beginning at some Knudsen number value, this change increases with decrease in Kn, i.e., as will be shown below, with increase in velocity u_L (Table 1). This is related to the fact that in the nonequilibrium layer referred to above two one-sided distribution functions "meet" each other, one of which considers the hydrodynamic resistance of the immobile particles. We will note that for small L/R the magnitude of this discontinuity is also large for large Kn. A phenomenological derivation of the boundary conditions on the porosity discontinuities was presented in [15, 16]. We note that in the present case, in contrast to those studies, the density in one of the media n_L is fixed quantity up to the very boundary x = 1. The presence of the density discontinuity is the result of the effect on mass transport of the boundary conditions at x = 1.

Using Eq. (19) and the expression for C obtained by solving the algebraic system, we find

$$u_L = -\frac{\varepsilon^2 \nu_L}{(4 - 2\varepsilon - \varepsilon^2 + A)\pi^{1/2}}. \tag{21}$$

In a similar manner we can solve the problem of gas filtration through a porous body when the gas density at the input (x = 0) n_0 and output (x = 1) n_L are known. In that case Eq. (11) must satisfy different boundary conditions. A general solution will not be presented, but we will note that at L/R >> 1 the filtration rate (like the evaporation rate) has the form

$$u_L = -\frac{\varepsilon^2 \nu_L}{\pi^{1/2} A} = -\frac{\pi^{1/2} \varepsilon^2 \nu_L R}{9(1-\varepsilon) L \,\mathrm{Kn}\, A_1(\mathrm{Kn})}. \tag{22}$$

We note that the case of diffusion flow (F = 0) can be studied approximately by writing the collision integral on the right of Eq. (1) in the relaxation form $I(f) = (f_0 - f)/\tau$, where f_0 is a Maxwell function without mass velocity, $\tau = \lambda_{ef}/\bar{\xi}$, $\lambda_{ef}^{-1} = \lambda^{-1} + \lambda_d^{-1}$. It can easily be shown that the expression obtained in this case for the diffusion coefficient gives a value close to the values obtained from the relationships derived in [17] based on the theory of mean molecular free path length with the assumption that interaction of gas molecules with each other and with the spheres is of identical character. We will discuss the accuracy of this approximation below, noting meanwhile that the mean molecular free path length permits study of more complex problems of this type (with consideration of adsorption, heterogeneous reactions, particle evaporation) [17-19] in regimes close to free molecular.

We will compare u_L values obtained in the free molecular regime (Kn → ∞) with available theoretical results. If we define the permeability $\varkappa$ from the relationship

TABLE 2. Calculated [Eq. (26)] and Experimental [22] Values of Permeability $\varkappa$ vs Kn for Catalyzer "B" for Hydrogen and Nitrogen Filtration

Kn	$\varkappa \cdot 10$, cm^2/sec; H_2		Kn	$\varkappa \cdot 10^2$, cm^2/sec; N_2	
	expt.	theory		expt.	theory
∞		0,333	∞		0,893
85,5	0,344	0,334	45,0	0,977	0,901
61,9	0,341	0,335	31,9	0,986	0,905
40,8	0,347	0,336	21,2	0,951	0,911
30,7	0,342	0,337	16,1	0,973	0,916
24,6	0,353	0,338	12,9	0,940	0,921
20,1	0,337	0,339			

$$nu_L\left(\frac{2kT}{m}\right)^{1/2} = -\varkappa\frac{dn}{dX}, \tag{23}$$

then from Eq. (21) as Kn $\to \infty$ we have

$$\varkappa_\infty = a\left(\frac{2kT}{m}\right)^{1/2}\frac{\varepsilon^2 d}{1-\varepsilon}, \tag{24}$$

where the numerical factor $a = \pi^{1/2}/(8+\pi) \approx 0.159$. A similar expression for $\varkappa_\infty$ follows from [1], with $a \approx 0.171$, and from [20], where $a \approx 0.122$ with consideration of the correction factor q (the convolution coefficient) introduced upon comparison with experimental data in the case of low porosity values $\varepsilon \approx 0.4$ (without this factor $a \approx 0.172$). We will also note that the mean molecular free path length referred to above gives a value of $a \approx 0.251$. The same result follows from [1] for the case of mirror reflection of molecules from the particle surface, which is equivalent to elastic interaction of molecules among themselves [21].

We will now turn to a comparison of the results obtained with the experimental data of [22]*, in which the permeability of porous catalyzers was measured. A generalized expression was presented in [22] for the gas flow N(r) in a unit cylindrical capillary of radius r in a porous body, with the aid of the equality $N = \psi N(r)$. It was assumed that, generally speaking, mass transport takes place only in certain pores, termed transport pores, i.e., $\varepsilon_T \leqslant \varepsilon$. The experiments used various gases and catalyzers (laboratory preparations of ZnO, CuO, Al_2O_3) at $20 \leqslant p \leqslant 190$ kPa, which achieved flow over a wide Knudsen number range.

In order to compare theoretical $\varkappa$ values with the experimental results of [22] for the case L/R >> 1, in Eq. (22) we must transform from particle radius R to pore radius r. According to [11], for a highly porous body the mean pore clearance $\lambda_d = 2r$ and the mean pore size d_m are related by the expression $\lambda_d = \varepsilon d_m/(1-\varepsilon)$, where for spheres $d_m = {}^2/_3 d$, for round cylinders $d_m = d$. Taking $d_m = d$, we find

$$R = \frac{1-\varepsilon}{\varepsilon}r. \tag{25}$$

For one of the catalyzers studied in [22] (denoted as catalyzer "B") practically all pores were transport pores ($\varepsilon_T = \varepsilon \approx 0.70$) and had approximately equal radii. For this catalyzer, we find from Eqs. (22), (23), (25)

$$\varkappa = \left(\frac{2kT}{m}\right)^{1/2}\frac{\pi^{1/2}\varepsilon r}{9\,\mathrm{Kn}\,A_1(\mathrm{Kn})}. \tag{26}$$

Substituting in Eq. (26) the effective pore radius r and porosity ε determined experimentally in [22], we can compare the results with the experimental $\varkappa$ values (we note that in [22] $\varkappa$ and ε were measured independently). In performing the comparison it should be kept in mind that Kn and the parameter $K = \lambda/2r$ used in [22] are related, according to Eq. (25), by the expression $\mathrm{Kn} = 2\varepsilon K/(1-\varepsilon)$. From Table 2 it is evident that $\varkappa_{ex}$ and $\varkappa$ coincide with good accuracy.

*In [22] the experimental data were only presented graphically, but numerical experimental results (also for a larger number of catalyzers) were kindly provided us by P. Fott (Institute of Theoretical Chemistry, Czechoslovak Academy of Sciences, Prague), to whom the authors expressed their gratitude.

TABLE 3. Calculated [Eq. (27)] and Experimental [22] Values of $\varkappa/\varkappa_\infty$ vs Kn for Catalyzers "A" and "D" for Nitrogen Filtration

Kn	$\varkappa/\varkappa_\infty$, «A»		Kn	$\varkappa/\varkappa_\infty$, «D»	
	expt.	theory		expt.	theory
∞		1	∞		1
13,91	1,081	1,029	5,46	1,023	1,072
10,28	1,057	1,039	3,98	1,021	1,096
6,71	1,061	1,059	2,56	1,061	1,143
5,06	1,043	1,077	1,93	1,137	1,183
4,06	1,088	1,094	1,54	1,152	1,226
3,34	1,064	1,112	1,28	1,103	1,270

For the other catalyzers the permeability $\varkappa$ is described by an expression analogous to Eq. (26), but containing two porosities, ε and ε_t. The experiments did not determine ε_t directly, but rather $\psi = \varepsilon_t/q$, with only possible limiting values being known for q. Thus for these catalyzers it is more convenient to compare values of the ratio $\varkappa/\varkappa_\infty$ with experiment:

$$\frac{\varkappa}{\varkappa_\infty} = \frac{[\mathrm{Kn}\,A_1(\mathrm{Kn})]_{\mathrm{Kn}\to\infty}}{\mathrm{Kn}\,A_1(\mathrm{Kn})} = \frac{8+\pi}{18\,\mathrm{Kn}\,A_1(\mathrm{Kn})}. \qquad (27)$$

These calculated $\varkappa/\varkappa_\infty$ values can be compared with experimental data for the quantity $\varkappa_{exp}/D_K$, where $D_K = {}^2/_3 r\psi\bar{\xi}$ is the effective diffusion coefficient in the porous body, determined from experimentally determined r and ψ values. These values are comparable because according to expressions presented in [22], $\varkappa \to D_K$ as $K \to \infty$. It is evident from Table 3 that for catalyzers "A" ($\varepsilon = 0.74$) and "D" ($\varepsilon = 0.64$) the agreement is completely satisfactory.

Thus, the theoretical results obtained and the corresponding experimental data (Tables 1-3) indicate that curves of u_L as a function of 1/Kn (Fig. 2) initially increase very slowly from their free molecular values. Then, approximately at Kn = 1, according to Eq. (21), a more abrupt increase in u_L begins, leading to a linear dependence $u_L \sim 1/\mathrm{Kn}$, characteristic of a viscous gas flow. Finally, with further increase in Kn in the case of evaporation u_L exits to a constant value, dependent solely on ε and ν_L:

$$u_L = -\frac{\varepsilon^2\nu_L}{(4-2\varepsilon-\varepsilon^2)\pi^{1/2}},$$

which as $\varepsilon \to 1$ tends to $u_{L_0} = -\nu_L/\pi^{1/2}$, which corresponds to evaporation from an open surface. We note that in the limit $L \to 0$ Eq. (21) for $\varepsilon \neq 1$ does not lead to the value $u_L = \varepsilon u_{L_0}$, corresponding to evaporation from a perforated surface; this apparently is a reflection of the error of the moment method in the given limiting case. In the filtration problem u_L does not exit to a constant value, and the dependence $u_L(\mathrm{Kn})$ found here is supported qualitatively by the experimental data of [20], obtained for $\varepsilon \approx 0.4$, for all Kn.

It follows from the above that Eqs. (21), (22) can be used to calculate the evaporation rate and mass transport through a porous layer of arbitrary thickness in the range $1 \leqslant \mathrm{Kn} \leqslant \infty$, i.e., in free molecular and intermediate gas flow regimes. As for the range of low Kn, here coincidence of experimental and calculated results can be expected only for sufficiently thin layers. In the general case of low Kn one must introduce into the permeability $\varkappa$ in the linear portion of the function $u_L(1/\mathrm{Kn})$ a correction coefficient $W_0(\varepsilon)$ [23, 24], which is dependent on ε and considers the "crowding" of the flow in the porous medium. Since according

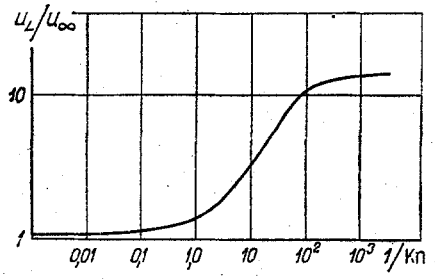

Fig. 2. Value of u_L normalized to u_L as $\mathrm{Kn} \to \infty(u_\infty)$, vs 1/Kn for evaporation from porous body with $\varepsilon = 0.9$, L/R = 100.

to Eq. (22) in the continuous medium regime (Kn → 0) the permeability $\varkappa_0 = \varepsilon^2 d^2 p/18(1-\varepsilon)\mu$ (similar expressions also follow from [2, 11]), the with consideration of $W_0(\varepsilon)$ we have:

$$\varkappa_0 = \frac{\varepsilon^2 d^2 p}{18(1-\varepsilon)\mu} W_0(\varepsilon),$$

where, for example, for $\varepsilon = 0.9$ the quantity $W = \varepsilon^2 W_0(\varepsilon) \approx 0.32$. A table of $W(\varepsilon)$ values was presented in [23], which were compared with experimental results. In particular, it was shown tht the Carmen–Kozeny expression for permeability (having the form $\varepsilon^3 d^2 p/180(1-\varepsilon)^2\mu$ in our notation) is applicable over the range $0.26 < \varepsilon < 0.7$.

Comparison with studies of gas filtration in which an expression for the gas flow valid over the entire range of Knudsen number change was used, written in the form of a superposition of Knudsen an viscous flows, shows that the greatest quantitative divergence of results occurs in the intermediate Knudsen number range. Thus, at Kn = 3 the respective relative filtration coefficients k/k_∞ [11] and $\varkappa/\varkappa_\infty$ differ by approximately 16%.

NOTATION

P_0, saturated vapor pressure; ε, porosity; λ, mean molecular free path length; Kn = λ/r, Knudsen number; d = 2R, particle radius; k, Boltzman's constant; m, gas molecule mass; $\rho = mn$, gas density; v, gas flow velocity in porous body; n_d, number of particles per unit volume of porous body; x = X/L, dimensionless coordinate; μ, kinematic gas viscosity.

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NONSTEADY UNIDIRECTIONAL DISCHARGE OF AN INSTANTANEOUSLY HEATED GAS WITH CONSTANT FORCED FLOW FROM A CYLINDER

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UDC 533.17:536.414

A numerical study is made of the problem of unidirectional discharge of an instantaneously heated gas from a half-open cylinder when the gas is pumped through the cylinder in the direction of the open end.

Calculation of thermohydrodynamic fields in half-open systems is of considerable interest for through-type electric-discharge quantum generators, where the uniformity of the medium has a significant effect on the working parameters of the system.

Apart from the specifics of the design of the system, the essence of the hydrodynamic process which accompanies the pressure jump in the working volume of the generator resulting from the discharge can be remodeled by the classical problem of unidirectional discharge of an instantaneously heated gas from a cylinder with one end open to the atmosphere under finite pressure [1]. It was shown in the solution of this problem that the initial pressure jump is accompanied by nonlinear oscillations of an amplitude which decreases slowly relative to the characteristic time scale. This result is in qualitative agreement with the test data obtained in [2] on a rarefaction wave tube. The slow return of uniformity in a system based on the principle of periodic-impulsive action when the only damping source is the local resistance at the open end of the cylinder (the friction against the walls has almost no effect on damping) stimulates searches for additional means of influencing the system parameters — one of which may be pumping the gas through the cylinder.

The present article studies gasdynamic processes in a cylindrical volume. One end of the cylinder is connected to the atmosphere. The gasdynamic processes are initiated by instantaneous heating of a gas in some middle section of the cylinder. We will study how the processes are affected by pumping the gas along the cylinder axis in the direction of the open end.

As usual in the gasdynamics of rapidly occuring processes, we will assume that the phenomena of viscosity, heat conduction, and external heat exchange have a slight effect on the

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 0022-0841/84/4606-0656$08.50